Introduction to Information Retrieval

http://informationretrieval.org

IIR 11: Probabilistic Information Retrieval

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2011-08-29
Models and Methods

1. Boolean model and its limitations (30)
2. Vector space model (30)
3. Probabilistic models (30)
4. Language model-based retrieval (30)
5. Latent semantic indexing (30)
6. Learning to rank (30)
Take-away
Take-away

- Probabilistic approach to IR: Introduction
Take-away

- **Probabilistic approach to IR**: Introduction
- **Binary independence model** or BIM – the first influential probabilistic model
Take-away

- Probabilistic approach to IR: Introduction
- Binary independence model or BIM – the first influential probabilistic model
- Okapi BM25, a more modern, better performing probabilistic model
Outline

1. Probabilistic Approach to IR
2. Binary independence model
3. Okapi BM25
The adhoc retrieval problem: Given a user information need and a collection of documents, the IR system must determine how well the documents satisfy the query.
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Probability theory provides a principled foundation for such reasoning under uncertainty.
Probabilistic approach to IR

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- The IR system has an uncertain understanding of the user query . . .
- . . . and makes an uncertain guess of whether a document satisfies the query.
- Probability theory provides a principled foundation for such reasoning under uncertainty.
- Probabilistic IR models exploit this foundation to estimate how likely it is that a document is relevant to a query.
Probabilistic vs. vector space model
Vector space model: rank documents according to similarity to query.
Probabilistic vs. vector space model

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- The notion of similarity does not translate directly into an assessment of “is the document a good document to give to the user or not?”
Probabilistic vs. vector space model

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- The most similar document can be highly relevant or completely nonrelevant.
Probabilistic vs. vector space model

- Vector space model: rank documents according to similarity to query.
- The notion of similarity does not translate directly into an assessment of “is the document a good document to give to the user or not?”
- The most similar document can be highly relevant or completely nonrelevant.
- Probability theory is arguably a cleaner formalization of what we really want an IR system to do: give relevant documents to the user.
Probabilistic Approach to IR

Probabilistic IR models at a glance
Probabilistic IR models at a glance

- Classical probabilistic retrieval models
Probabilistic IR models at a glance

- Classical probabilistic retrieval models
  - Binary Independence Model
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Probabilistic IR models at a glance

- Classical probabilistic retrieval models
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- Bayesian networks for text retrieval
  - Don’t have time for this
- Language model approach to IR
  - Important recent work, will be covered in the next lecture
Ranked retrieval setup: the user issues a query, and a ranked list of documents is returned.
Probabilistic IR and ranking

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- (This is a binary notion of relevance.)
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- Probabilistic ranking orders documents decreasingly by their estimated probability of relevance w.r.t. query: $P(R = 1|d, q)$
- How can we justify this way of proceeding?
Probability Ranking Principle (PRP)

If the retrieved documents are ranked decreasingly on their probability of relevance (w.r.t a query), then the effectiveness of the system will be the best that is obtainable.
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If the retrieved documents are ranked decreasingly on their probability of relevance (w.r.t a query), then the effectiveness of the system will be the best that is obtainable.

Fundamental assumption: the relevance of each document is independent of the relevance of other documents.
Outline

1. Probabilistic Approach to IR

2. Binary independence model

3. Okapi BM25
Binary Independence Model (BIM)

- **Binary**: documents and queries represented as binary term incidence vectors
Binary Independence Model (BIM)

- **Binary**: documents and queries represented as binary term incidence vectors
- **Independence**: terms are independent of each other (not true, but works in practice – naive assumption of Naive Bayes models)
## Binary incidence matrix

<table>
<thead>
<tr>
<th></th>
<th>Anthony and Cleopatra</th>
<th>Julius Caesar</th>
<th>The Tempest</th>
<th>Hamlet</th>
<th>Othello</th>
<th>Macbeth</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ANTHONY</strong></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>BRUTUS</strong></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>CAESAR</strong></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>CALPURNIA</strong></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>CLEOPATRA</strong></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>MERCY</strong></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>WORSER</strong></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Each document is represented as a binary vector $\in \{0, 1\}^{|V|}$.  

□
Bayes’ rule
Bayes’ rule

\[
P(R = 1|\vec{x}, \vec{q}) = \frac{P(\vec{x}|R = 1, \vec{q})P(R = 1|\vec{q})}{P(\vec{x}|\vec{q})}
\]

\[
P(R = 0|\vec{x}, \vec{q}) = \frac{P(\vec{x}|R = 0, \vec{q})P(R = 0|\vec{q})}{P(\vec{x}|\vec{q})}
\]
Bayes’ rule

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\]

(Recall that document and query are modeled as term incidence vectors: \(\vec{x}\) and \(\vec{q}\).)
Bayes’ rule

\[ P(R = 1|\vec{x}, \vec{q}) = \frac{P(\vec{x}|R = 1, \vec{q})P(R = 1|\vec{q})}{P(\vec{x}|\vec{q})} \]

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(Recall that document and query are modeled as term incidence vectors: \( \vec{x} \) and \( \vec{q} \).)

\( P(\vec{x}|R = 1, \vec{q}) \) and \( P(\vec{x}|R = 0, \vec{q}) \): probability that if a relevant or nonrelevant document is retrieved, then that document’s representation is \( \vec{x} \)
Bayes’ rule

\[ P(R = 1 | \vec{x}, \vec{q}) = \frac{P(\vec{x} | R = 1, \vec{q}) P(R = 1 | \vec{q})}{P(\vec{x} | \vec{q})} \]

\[ P(R = 0 | \vec{x}, \vec{q}) = \frac{P(\vec{x} | R = 0, \vec{q}) P(R = 0 | \vec{q})}{P(\vec{x} | \vec{q})} \]

(Recall that document and query are modeled as term incidence vectors: \( \vec{x} \) and \( \vec{q} \).)

- \( P(\vec{x} | R = 1, \vec{q}) \) and \( P(\vec{x} | R = 0, \vec{q}) \): probability that if a relevant or nonrelevant document is retrieved, then that document’s representation is \( \vec{x} \)

- Use statistics about the document collection to estimate these probabilities
Priors

$P(R|d, q)$ is modeled using term incidence vectors as $P(R|x, q)$

\[
P(R = 1|x, q) = \frac{P(x|R = 1, q)P(R = 1|q)}{P(x|q)}
\]
\[
P(R = 0|x, q) = \frac{P(x|R = 0, q)P(R = 0|q)}{P(x|q)}
\]
Priors

\( P(R|d, q) \) is modeled using term incidence vectors as \( P(R|\vec{x}, \vec{q}) \)

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P(R = 1|\vec{x}, \vec{q}) = \frac{P(\vec{x}|R = 1, \vec{q})P(R = 1|\vec{q})}{P(\vec{x}|\vec{q})}
\]

\[
P(R = 0|\vec{x}, \vec{q}) = \frac{P(\vec{x}|R = 0, \vec{q})P(R = 0|\vec{q})}{P(\vec{x}|\vec{q})}
\]

- \( P(R = 1|\vec{q}) \) and \( P(R = 0|\vec{q}) \): prior probability of retrieving a relevant or nonrelevant document for a query \( \vec{q} \)
Priors

\( P(R|d, q) \) is modeled using term incidence vectors as \( P(R|x, \tilde{q}) \)

\[
\begin{align*}
P(R = 1|x, \tilde{q}) &= \frac{P(x|R = 1, \tilde{q})P(R = 1|\tilde{q})}{P(x|\tilde{q})} \\
P(R = 0|x, \tilde{q}) &= \frac{P(x|R = 0, \tilde{q})P(R = 0|\tilde{q})}{P(x|\tilde{q})}
\end{align*}
\]

- \( P(R = 1|\tilde{q}) \) and \( P(R = 0|\tilde{q}) \): prior probability of retrieving a relevant or nonrelevant document for a query \( \tilde{q} \)
- Estimate \( P(R = 1|\tilde{q}) \) and \( P(R = 0|\tilde{q}) \) from percentage of relevant documents in the collection
We said that we’re going to rank documents according to 
\[ P(R = 1|\vec{x}, \vec{q}) \]
Ranking according to odds

- We said that we’re going to rank documents according to $P(R = 1|\vec{x}, \vec{q})$
- Easier: rank documents by their odds of relevance (gives same ranking)

$$O(R|\vec{x}, \vec{q}) = \frac{P(R = 1|\vec{x}, \vec{q})}{P(R = 0|\vec{x}, \vec{q})} = \frac{P(R=1|\vec{q})P(\vec{x}|R=1,\vec{q})}{P(\vec{x}|\vec{q})} \cdot \frac{P(R=0|\vec{q})P(\vec{x}|R=0,\vec{q})}{P(\vec{x}|\vec{q})}$$

$$= \frac{P(R = 1|\vec{q})}{P(R = 0|\vec{q})} \cdot \frac{P(\vec{x}|R = 1, \vec{q})}{P(\vec{x}|R = 0, \vec{q})}$$
We said that we’re going to rank documents according to
\[ P(R = 1 | \bar{x}, \bar{q}) \]
Easier: rank documents by their odds of relevance (gives same ranking)
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O(R | \bar{x}, \bar{q}) = \frac{P(R = 1 | \bar{x}, \bar{q})}{P(R = 0 | \bar{x}, \bar{q})} = \frac{\frac{P(R=1|\bar{q})P(\bar{x}|R=1,\bar{q})}{P(\bar{x}|\bar{q})}}{\frac{P(R=0|\bar{q})P(\bar{x}|R=0,\bar{q})}{P(\bar{x}|\bar{q})}} = \frac{P(R = 1 | \bar{q})}{P(R = 0 | \bar{q})} \cdot \frac{P(\bar{x}|R = 1, \bar{q})}{P(\bar{x}|R = 0, \bar{q})}
\]
\[ \frac{P(R=1|\bar{q})}{P(R=0|\bar{q})} \] is a constant for a given query - can be ignored
Naive Bayes conditional independence assumption
Now we make the **Naive Bayes conditional independence assumption** that the presence or absence of a word in a document is independent of the presence or absence of any other word (given the query):

\[
\frac{P(\vec{x} | R = 1, \vec{q})}{P(\vec{x} | R = 0, \vec{q})} = \frac{\prod_{t=1}^{M} P(x_t | R = 1, \vec{q})}{\prod_{t=1}^{M} P(x_t | R = 0, \vec{q})}
\]

So:

\[
O(R | \vec{x}, \vec{q}) \propto \prod_{t=1}^{M} \frac{P(x_t | R = 1, \vec{q})}{P(x_t | R = 0, \vec{q})}
\]
Separating terms in the document vs. not

Since each $x_t$ is either 0 or 1, we can separate the terms:
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$$O(R|\vec{x}, \vec{q}) \propto \prod_{t: x_t = 1} \frac{P(x_t = 1|R = 1, \vec{q})}{P(x_t = 1|R = 0, \vec{q})} \prod_{t: x_t = 0} \frac{P(x_t = 0|R = 1, \vec{q})}{P(x_t = 0|R = 0, \vec{q})}$$
Definition of $p_t$ and $u_t$

- Let $p_t = P(x_t = 1|R = 1, \vec{q})$ be the probability of a term appearing in relevant document.
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- Can be displayed as contingency table:

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<td>$p_t$</td>
</tr>
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$$O(R|\vec{x}, \vec{q}) \propto \prod_{t: x_t = 1} \frac{p_t}{u_t} \prod_{t: x_t = 0} \frac{1 - p_t}{1 - u_t}$$
Dropping terms that don’t occur in the query
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- Additional simplifying assumption: If $q_t = 0$, then $p_t = u_t$
Dropping terms that don’t occur in the query

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- A term not occurring in the query is equally likely to occur in relevant and nonrelevant documents.
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O(R|\vec{x}, \vec{q}) \propto \prod_{t: x_t = 1} \frac{p_t}{u_t} \prod_{t: x_t = 0} \frac{1 - p_t}{1 - u_t} \approx \prod_{t: x_t = q_t = 1} \frac{p_t}{u_t} \prod_{t: x_t = 0, q_t = 1} \frac{1 - p_t}{1 - u_t}
\]
BIM retrieval status value
BIM retrieval status value

Including the query terms found in the document into the right product, but simultaneously dividing by them in the left product, gives:

\[ O(R|\vec{x}, \vec{q}) \propto \prod_{t:x_t=q_t=1} \frac{p_t(1-u_t)}{u_t(1-p_t)} \cdot \prod_{t:q_t=1} \frac{1-p_t}{1-u_t} \]
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- The right product is now over all query terms, hence constant for a particular query and can be ignored.
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- The right product is now over all query terms, hence constant for a particular query and can be ignored.

\[ \rightarrow \text{The only quantity that needs to be estimated to rank documents w.r.t a query is the left product.} \]

- Hence the Retrieval Status Value (RSV) in this model:

\[ RSV_d = \log \prod_{t: x_t=q_t=1} \frac{p_t(1-u_t)}{u_t(1-p_t)} = \sum_{t: x_t=q_t=1} \log \frac{p_t(1-u_t)}{u_t(1-p_t)} \]
BIM retrieval status value (2)
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Equivalent: rank documents using the log odds ratios for the terms in the query \( c_t \):

\[
c_t = \log \frac{p_t(1 - u_t)}{u_t(1 - p_t)} = \log \frac{p_t}{1 - p_t} - \log \frac{u_t}{1 - u_t}
\]

The odds ratio is the ratio of two odds: (i) the odds of the term appearing if the document is relevant \((p_t/(1 - p_t))\), and (ii) the odds of the term appearing if the document is nonrelevant \((u_t/(1 - u_t))\).
BIM retrieval status value (2)

Equivalent: rank documents using the log odds ratios for the terms in the query $c_t$:

$$c_t = \log \frac{p_t(1 - u_t)}{u_t(1 - p_t)} = \log \frac{p_t}{(1 - p_t)} - \log \frac{u_t}{1 - u_t}$$

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- $c_t = 0$: term has equal odds of appearing in relevant and nonrelevant docs
- $c_t$ positive: higher odds to appear in relevant documents
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- $c_t$ positive: higher odds to appear in relevant documents

- $c_t$ negative: higher odds to appear in nonrelevant documents
Term weight $c_t$ in BIM

$$c_t = \log \frac{p_t}{1-p_t} - \log \frac{u_t}{1-u_t}$$ functions as a term weight.
Term weight $c_t$ in BIM

- $c_t = \log \frac{p_t}{(1-p_t)} - \log \frac{u_t}{1-u_t}$ functions as a term weight.
- Retrieval status value for document $d$: $RSV_d = \sum_{x_t=q_t=1} c_t$. 
Term weight $c_t$ in BIM

- $c_t = \log \frac{p_t}{(1-p_t)} - \log \frac{u_t}{1-u_t}$ functions as a term weight.
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- So BIM and vector space model are similar on an operational level.
- In particular: we can use the same data structures (inverted index etc) for the two models.
Computing term weights $c_t$

For each term $t$ in a query, estimate $c_t$ in the whole collection using a contingency table of counts of documents in the collection, where $df_t$ is the number of documents that contain term $t$:

<table>
<thead>
<tr>
<th>documents</th>
<th>relevant</th>
<th>nonrelevant</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term present</td>
<td>$x_t = 1$</td>
<td>$s$</td>
<td>$df_t - s$</td>
</tr>
<tr>
<td>Term absent</td>
<td>$x_t = 0$</td>
<td>$S - s$</td>
<td>$(N - df_t) - (S - s)$</td>
</tr>
<tr>
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$$p_t = \frac{s}{S}$$

$$u_t = \frac{(df_t - s)}{(N - S)}$$

$$c_t = K(N, df_t, S, s) = \log \frac{s/(S - s)}{(df_t - s)/((N - df_t) - (S - s))}$$
Avoiding zeros
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- If any of the counts is a zero, then the term weight is not well-defined.
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- To avoid zeros: add 0.5 to each count (expected likelihood estimation = ELE) or use a different type of smoothing.
More simplifying assumptions
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- ... then we can approximate statistics for nonrelevant documents by statistics from the whole collection:

$$\log[(1 - u_t)/u_t] = \log[(N - df_t)/df_t] \approx \log N/df_t$$
More simplifying assumptions

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  \]
- This should look familiar to you ...
Probability estimates in relevance feedback
For relevance feedback, we can directly compute term weights $c_t$ based on the contingency table (using an appropriate smoothing method like ELE).
Computing term weights $c_t$ for relevance feedback

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- For short documents (titles or abstracts), this simple version of BIM works well.
Outline

1. Probabilistic Approach to IR
2. Binary independence model
3. Okapi BM25
Okapi BM25 is a probabilistic model that incorporates term frequency (i.e., it’s nonbinary) and length normalization.
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- For modern full-text search collections, a model should pay attention to term frequency and document length.
- BM25 (BestMatch25) is sensitive to these quantities.
Okapi BM25: Starting point
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Probabilistic approach to IR: Introduction

Binary independence model or BIM – the first influential probabilistic model

Okapi BM25, a more modern, better performing probabilistic model
Resources

- Chapter 11 of Introduction to Information Retrieval
- Resources at http://informationretrieval.org/essir2011
  - Binary independence model (original paper)
  - More details on Okapi BM25
  - Why the Naive Bayes independence assumption often works (paper)
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