Introduction to Information Retrieval

http://informationretrieval.org

IIR 18: Latent Semantic Indexing

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Models and Methods

1. Boolean model and its limitations (30)
2. Vector space model (30)
3. Probabilistic models (30)
4. Language model-based retrieval (30)
5. Latent semantic indexing (30)
6. Learning to rank (30)

Take-away

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- Singular Value Decomposition (SVD): The math behind LSI

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- Singular Value Decomposition (SVD): The math behind LSI
- SVD used for dimensionality reduction
- Latent Semantic Indexing (LSI): SVD used in information retrieval

Outline

1. Singular Value Decomposition
2. Dimensionality reduction
3. Latent Semantic Indexing

Recall: Term-document matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
anthony	5.25	3.18	0.0	0.0	0.0	0.35
brutus	1.21	6.10	0.0	1.0	0.0	0.0
caesar	8.59	2.54	0.0	1.51	0.25	0.0
calpurnia	0.0	1.54	0.0	0.0	0.0	0.0
cleopatra	2.85	0.0	0.0	0.0	0.0	0.0
mercy	1.51	0.0	1.90	0.12	5.25	0.88
worser	1.37	0.0	0.11	4.15	0.25	1.95
...						

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caesar	8.59	2.54	0.0	1.51	0.25	0.0
calpurnia	0.0	1.54	0.0	0.0	0.0	0.0
cleopatra	2.85	0.0	0.0	0.0	0.0	0.0
mercy	1.51	0.0	1.90	0.12	5.25	0.88
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This matrix is the basis for computing the similarity between documents and queries.

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This lecture: Can we transform this matrix, so that we get a better measure of similarity between documents and queries? □

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- We'll get better similarity values out of C' (compared to C).
- Using SVD for this purpose is called latent semantic indexing or LSI. □

Example of $C = U\Sigma V^T$: The matrix C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

This is a standard term-document matrix.

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ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

This is a standard term-document matrix.

Actually, we use a non-weighted matrix here to simplify the example.

Example of $C = U\Sigma V^T$: The matrix U

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

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Each number u_{ij} in the matrix indicates how strongly related term i is to the topic represented by semantic dimension j. □

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Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

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We'll make use of this by omitting unimportant dimensions.

Example of $C = U\Sigma V^T$: The matrix V^T

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22
6	0.00	0.00	0.00	-0.58	0.58	0.58

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3	0.28	−0.75	0.45	−0.20	0.12	−0.33
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$N \times N$ square matrix.

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These are again the semantic dimensions from matrices U and Σ that capture distinct topics like politics, sports, economics.

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Example of $C = U \Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1.00	0.00	1.00	0.00	0.00	0.00
boat	0.00	1.00	0.00	0.00	0.00	0.00
ocean	1.00	1.00	0.00	0.00	0.00	0.00
wood	1.00	0.00	0.00	1.00	1.00	0.00
tree	0.00	0.00	0.00	1.00	0.00	1.00

$=$

U	1	2	3	4	5
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LSI is decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.

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- Next: Why are we doing this? □

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  - Image of a blue flower
  - Image of a yellow flower
  - Omitting color makes is easier to see the similarity

Reducing the dimensionality to 2

U	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
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Σ_2	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
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tree	−0.26	0.65	0.00	0.00	0.00

Σ_2	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
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1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
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3	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00

Actually, we only zero out singular values in Σ. This has the effect of setting the corresponding dimensions in U and V^T to zero when computing the product $C = U\Sigma V^T$. □

Reducing the dimensionality to 2

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	−0.08
boat	0.36	0.36	0.16	−0.20	−0.02	−0.18
ocean	1.01	0.72	0.36	−0.04	0.16	−0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	−0.39	−0.08	0.90	0.41	0.49

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

×

Σ_2	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00

×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22

Recall unreduced decomposition $C = U\Sigma V^T$

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1.00	0.00	1.00	0.00	0.00	0.00
boat	0.00	1.00	0.00	0.00	0.00	0.00
ocean	1.00	1.00	0.00	0.00	0.00	0.00
wood	1.00	0.00	0.00	1.00	1.00	0.00
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Original matrix C vs. reduced $C_2 = U\Sigma_2 V^T$

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

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We can view C_2 as a two-dimensional representation of the matrix C. We have performed a dimensionality reduction to two dimensions.

Why the reduced matrix C_2 is better than C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
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Similarity of d_2 and d_3 in the reduced space: $0.52 * 0.28 + 0.36 * 0.16 + 0.72 * 0.36 + 0.12 * 0.20 + -0.39 * -0.08 \approx 0.52$

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"boat" and "ship" are semantically similar. The "reduced" similarity measure reflects this. □

Outline

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- Standard vector space: Synonyms contribute nothing to document similarity.
- Desired effect of LSI: Synonyms contribute strongly to document similarity.

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- But it will avoid doing that for unrelated words. □

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- . . . and it has the same problems.

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- Exercise: What is the fundamental problem with this approach?

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- So LSI uses the "best possible" matrix.
- There is only one best possible matrix – unique solution (modulo signs).
- Caveat: There is only a tenuous relationship between the Frobenius norm and cosine similarity between documents. □

Data for graphical illustration of LSI

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c_1	Human machine interface for lab abc computer applications
c_2	A survey of user opinion of computer system response time
c_3	The EPS user interface management system
c_4	System and human system engineering testing of EPS
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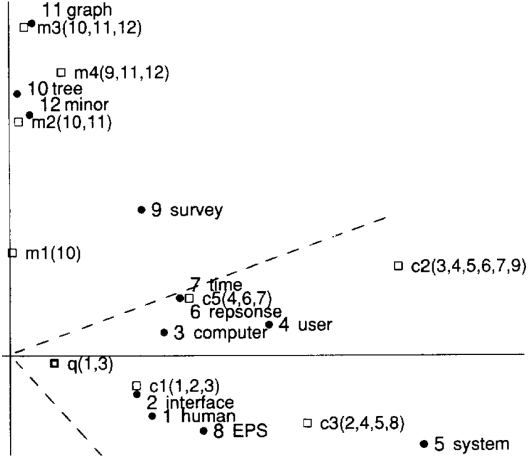
The matrix C

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

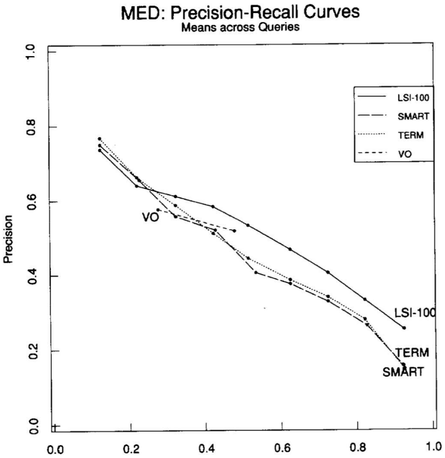
Graphical illustration of LSI: Plot of C_2



2-dimensional plot of C_2 (scaled dimensions). Circles = terms. Open squares = documents (component terms in parentheses). q = query "human computer interaction".

The dotted cone represents the region whose points are within a cosine of .9 from q . All documents about human-computer documents (c1-c5) are near q, even c3/c5 although they share no terms. None of the graph theory documents (m1-m4) are near q.

LSI performs better than vector space on MED collection



LSI-100 = LSI reduced to 100 dimensions; SMART = SMART implementation of vector space model

Take-away

- Singular Value Decomposition (SVD): The math behind LSI
- SVD used for dimensionality reduction
- Latent Semantic Indexing (LSI): SVD used in information retrieval

Resources

- Chapter 18 of Introduction to Information Retrieval
- Resources at http://informationretrieval.org/essir2011
  - Latent semantic indexing by Deerwester et al. (original paper)
  - Probabilistic LSI by Hofmann
  - Word space: LSI for words